Knowledge representation

lecture 4

Rules in logic with provers to answer questions

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Lecture overview

Rules and logic

Simple derivation systems for 1st order logic: resolution

Prolog as a special resolution search strategy

Queries and answers

Why rules

Derive new knowledge from existing knowledge

Learning: automatic creation of new rules

Inferences: apply rules to get new knowledge

Is the „derived knowledge" really new or just an intrinsic consequence of data + rules? This is a philosophical question out of the scope of this course.

Applying 1st order logic

There are many different - mostly equivalent - axiomatizations and rule systems for logic.

However, the practical - and sufficient - way to think is simply this:

- Find all possible matches with the premisses of the rule
- Each match instantiates variables
- Derive an instantiated consequence of the rule
- Repeat for all matches and all consequences etc etc ad infinitum

Conjunctive normal form

Goal: make a fact/rule set simple and uniform. Remove nested loops and existential quantifiers.

Terminology:

Atom is a propositional variable or a predicate with args like **p(X,1)**

Literal is a an atom or a negation of an atom;

Clause or disjunct is a disjunction of literals

Conjuctive normal form is a conjuction of clauses.

Convert to normal form

There are six stages to the conversion:

1. Remove $\Rightarrow$
2. De Morgan’s to move negation to atomic propositions
3. Skolemizing (gets rid of $\exists$)
4. `Eliminating’ universal quantifiers
5. Distributing AND over OR
6. Arrange into clauses and maybe reorder

Convert to normal form: example

1st order formula

$\forall Y\ (\forall X\ (\text{taller}(Y,X) \mid \text{wise}(X)) \Rightarrow \text{wise}(Y))$

Simplify

$\forall Y\ (-\forall X\ (\text{taller}(Y,X) \mid \text{wise}(X)) \mid \text{wise}(Y))$

Move negations in

$\forall Y\ (\exists X\ (-\text{taller}(Y,X)\ \&\ -\text{wise}(X)) \mid \text{wise}(Y))$

Move quantifiers out

$\forall Y\ (\exists X\ ((-\text{taller}(Y,X)\ \&\ -\text{wise}(X)) \mid \text{wise}(Y)))$

Skolemize

$\exists X\ ((-\text{taller}(Y,X)\ \&\ -\text{wise}(X)) \mid \text{wise}(Y))\ \gamma = \{Y\}$

$(-\text{taller}(Y,x(Y))\ \&\ -\text{wise}(x(Y))) \mid \text{wise}(Y)$

Distribute disjunctions

$(-\text{taller}(Y,x(Y)) \mid \text{wise}(Y))\ \&\ (-\text{wise}(x(Y)) \mid \text{wise}(Y))$

Convert to CNF

$\{\ -\text{taller}(Y,x(Y)) \mid \text{wise}(Y),\quad -\text{wise}(x(Y)) \mid \text{wise}(Y)\ \}$

Example: facts and rules in a normal form

father(john,pete).

brother(pete,mark).

-brother(X,Y) v brother(Y,X).

-father(X,Y) v parent(X,Y).

-mother(X,Y) v parent(X,Y).

-parent(X,Y) v -parent(Y,Z) v grandparent(X,Z).

Note: we assume capital letters are variables

Resolution method

Simple core method for practical reasoning (logical derivations)

Used as a basis for most reasoner implementations, with numerous additions / modifications / strategies / optimisations.

Can be seen as a framework for building specialized reasoners.

Just two rules operating on a normal form:

- Generalised modus ponens (**resolution rule**)
- Limited instantiation (**factorisation rule**)

The main idea of the the resolution method: derive new clauses from given clauses potentially ad infinitum.

Typically used to show that a clause set is **contradictory**.

To prove that F is a tautology is the same as to prove that -F is contradictory.

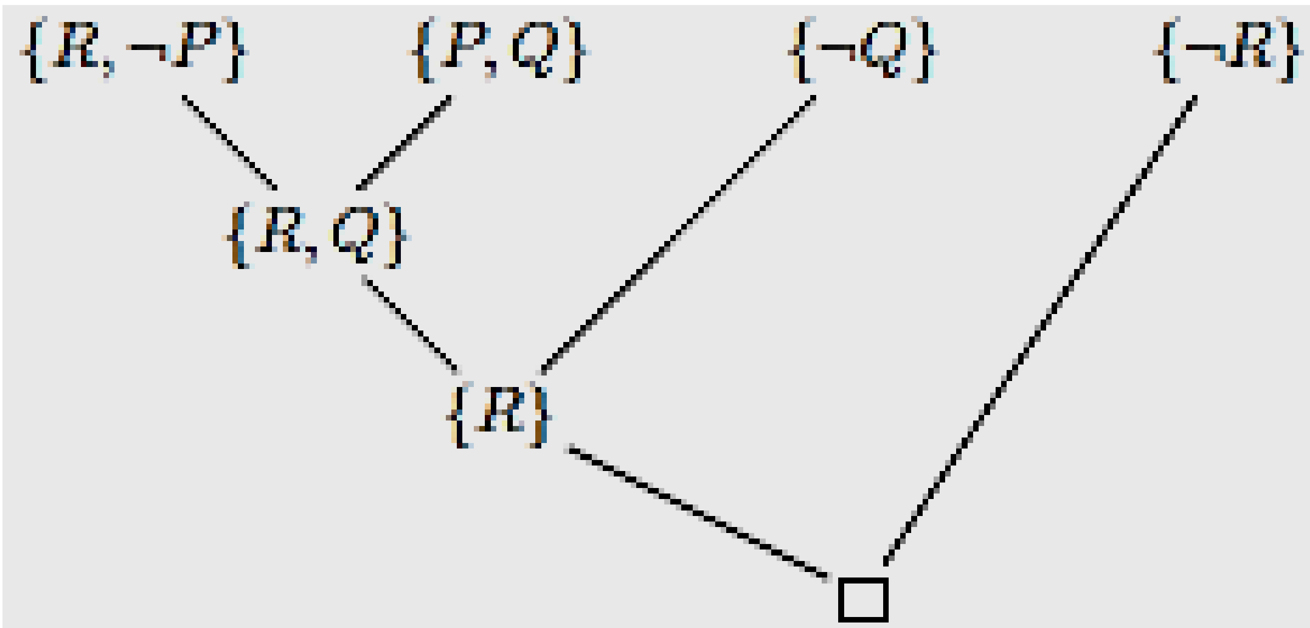
Modus ponens:

$$\frac{A \qquad A \rightarrow B}{B}$$

Same, but with A->B as a disjunct:

$$\frac{A \qquad -A \lor B}{B}$$

Example derivation of contradiction:



Resolution rule is a generalisation of modus ponens to arbitrary disjuncts:

$$\frac{\mathbf{A_1} \vee A_2 \vee \ldots \vee A_n \qquad \neg\mathbf{A_1} \vee B_2 \vee \ldots \vee B_m}{A_2 \vee \ldots \vee A_n \vee B_1 \vee \ldots \vee B_m}$$

How to apply the rule:

- Find a variable A_1 which is positive in one formula and negative in the ohter
- Cut off both A_1 and $\neg A_1$ and glue the rest.

Resolution rule for predicate calculus

Example (observe that P(b) is a different atom than P(X)):

P(b)   P(X) => R(X)

R(b)   vars instantiated X:=b

Resolution rule
for predicate calculus

Same example in normal form:

P(b)   -P(X) v R(X)

R(b)   vars instantiated X:=b

Resolution rule for predicate calculus

Example with additional ballast:

P(b) v G(s)   S(Y)   -P(X) v -S(X) v R(X) v M(X)

R(b) v M(b) v G(s)   vars instantiated X:=b, Y:=b

Resolution rule
for predicate calculus

Example with three premises

P(b)   S(Y)   P(X) & S(X) => R(X)

R(b)   vars instantiated X:=b, Y:=b

is the same as two steps of resolution:

P(b)   -P(X) v -S(X) v R(X)

-S(b) v R(b)   S(Y)

R(b)

Full rule with unification for predicate calculus:

$$\frac{\mathbf{A_1} \vee A_2 \vee \ldots \vee A_n \qquad \neg\mathbf{A_1} \vee B_2 \vee \ldots \vee B_m}{(A_2 \vee \ldots \vee A_n \vee B_1 \vee \ldots \vee B_m)s}$$

Where s = unify(A1,B1) :

unify(A,B) calculates *the most general unifier* of A and B: it is the minimal instantiation s making As=Bs

Unification examples:

P(X,a,Y),
P(Z,U,Z)) Gives {X:=Z, U:=a, Y:=Z }

p(X, f(cat))
p(f(Y) ,f(Y))
p(f(Z), T)

Gives
{X:=f(cat),Y:=cat,T:=f(cat),Z:=cat}

p(X, f(cat)),
p(f(Y), f(Y)),
p(f(dog), Z)}

Fails

p(f(Y), f(Y))
p(f(Z), Z)

Fails (occur check)

Factorization: eliminate duplicates

$$\frac{\mathbf{A_1} \vee \mathbf{A_1} \vee A_2 \vee \ldots \vee A_n}{\mathbf{A_1} \vee A_2 \vee \ldots \vee A_n}$$

Factorization for predicate calculus

Example:

P(X) v P(a)

P(a)

Rule for gluing together two literals in the same disjunct using the minimal unifier.

A1 v A2 v ... v An
and s = unify(A1,A2)

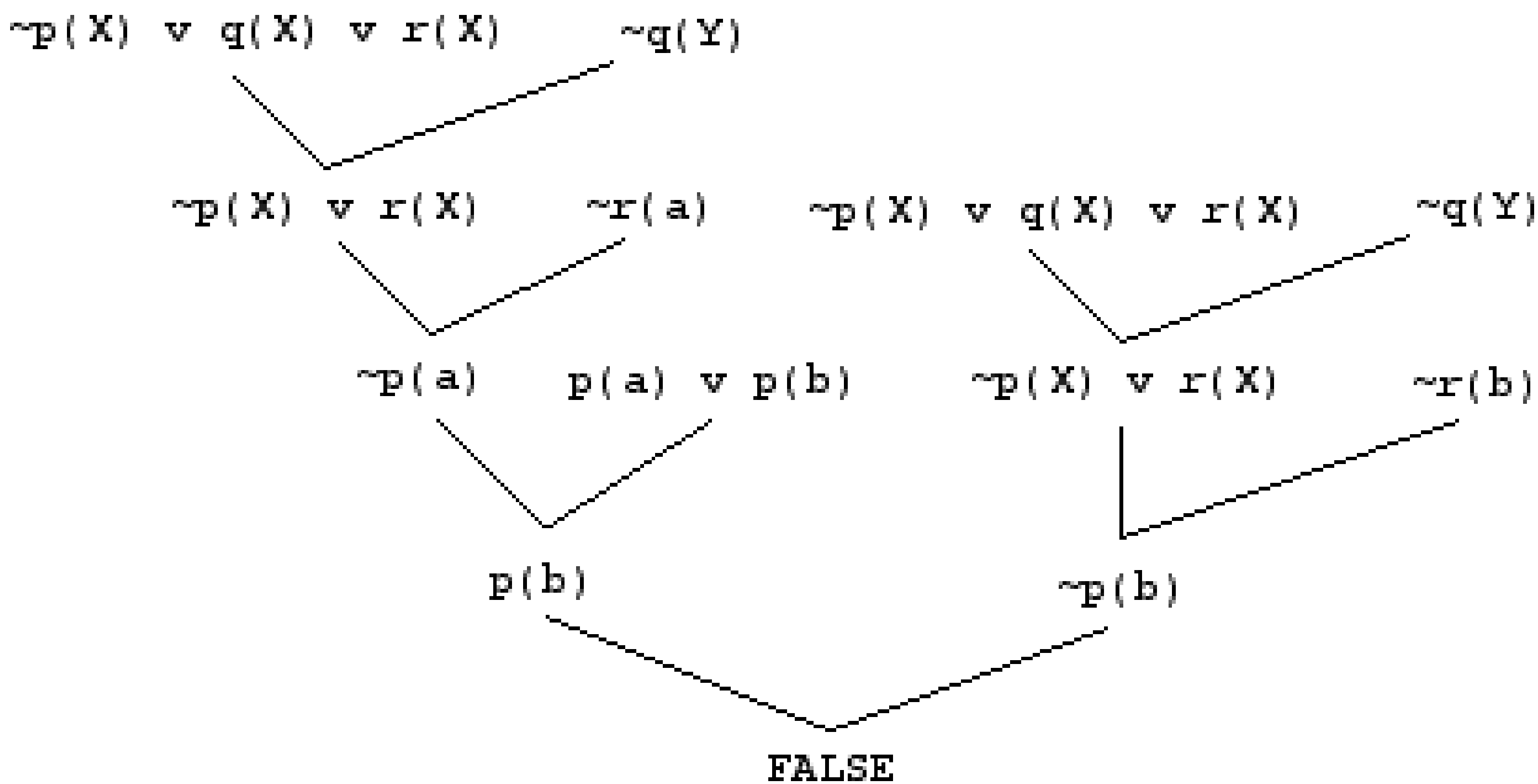
(A2 v ... v An) s

Basic saturation procedure

While a refutation has not been found:

- Copy two clauses from the set
- Generate all logical consequences, e.g., by resolution
- Put logical consequences into the set

Derivation example



ANL loop for saturation

Let CanBeUsed = {}

Let ToBeUsed = Input clauses

While Refutation not found && ToBeUsed not empty:

    Select the ChosenClause from ToBeUsed
    Move the ChosenClause to CanBeUsed
    Infer all possible clauses using the ChosenClause and other clauses
        from CanBeUsed.
    Add the inferred clauses to ToBeUsed

ANL loop for saturation

Depending on how the ChosenClause is selected from the ToBeUsed set, different search strategies can be implemented.

- **Depth first search**
  Select a most recently created resolvant as the ChosenClause.
  Does not guarantee a complete search (could get into an infinite loop)
  Does not guarantee finding the shortest refutation.

- **Breadth first search**
  Select a least recently created resolvant as the ChosenClause.
  Will find the shortest refutation.
  Implements a ply-by-ply search.

- **Best first search**
  Select the 'best' clause as the ChosenClause. the best possible literals.
  The notion of "best" is determined by a heuristic function

Saturation may not terminate

In case the clause set is not contradictory (i.e. empty clause cannot be derived) the saturation method may run forever

Simple example:

$\neg P(x) \lor P(f(x))$
$P(y) \lor \neg P(f(y))$

Will give $\neg P(x) \vee \neg P(f(f(x))$, $\neg P(x) \vee \neg P(f(f(f(x)))$, ... etc

However, for these clauses saturation will stop quickly:

$\neg P(x) \lor P(a)$
$P(y) \lor \neg P(b)$

Query: contradiction search

Observe that

Facts & Rules => Query

is a tautology iff

Facts & Rules & -Query

is a contradiction

Since -(Facts & Rules => Query)   =   -(-Facts v -Rules v Query)

Answer mechanism

Prolog query ? R(X) means adding
-R(X) | Answer(X)
to facts and then searching for contradiction:

-R(X) | Answer(X)
P(a)
-P(X) | R(X)

-P(X) | Answer(X)

Answer(a)

Prolog

Uses a highly specialised (and incomplete!) strategy of resolution for horn clauses: rules with max one literal in the consequent
(max one positive literal in a disjunct)

Derivation direction always from positive (right) side of the rule, giving instantiated antecedent as a result:

P(a)
P(X) => R(X)
-R(a) (query)
----
derives -P(a), and then finds contradiction with P(a)

Does not derive R(a).

Horn clause is a disjunct with max one positive literal.

-A v -B v C	is Horn
-A v -B	is Horn
-A v -B v C v D	not Horn
A v B	not Horn

Unit resolution is a search strategy of resoluton where at least one of arguments must be unit (a single literal).

Theorem: unit resolution is complete for Horn clause sets.

Subsumption:

A subset S of a disjunct D is said to subsume D.

Examples:

A subsumes –B v A

A v B subsumes C v B v –R v A

Theorem: when a derived disjunct N subsumes another disjunct C, then throwing away C preserves completeness

Complexity of unit resolution for the Horn case

Observe that every unit resolution rule application to Horn clauses creates a shorter disjunct which subsumes a long argument.

Example:

$$\frac{A \qquad\qquad -A \vee -B \;\; \vee C}{-B \vee C}$$

Robbins algebras are boolean: Mccune, 1997

In 1933, E. V. Huntington presented the following basis for Boolean algebra:

$x + y = y + x.$

$(x + y) + z = x + (y + z).$

$n(n(x) + y) + n(n(x) + n(y)) = x.$

Shortly thereafter, Herbert Robbins conjectured that the Huntington equation can be replaced with a simpler : $n(n(x + y) + n(x + n(y))) = x$. Robbins and Huntington could not find a proof, and the problem was later studied by Tarski and his students.

The successful search took about 8 days on an RS/6000 processor and used about 30 megabytes of memory.

2 (wt=7) [] -(n(x + y) = n(x)).

3 (wt=13) [] n(n(n(x) + y) + n(x + y)) = y.

5 (wt=18) [para(3,3)] n(n(n(x + y) + n(x) + y) + y) = n(x + y).

6 (wt=19) [para(3,3)] n(n(n(n(x) + y) + x + y) + y) = n(n(x) + y).

24 (wt=21) [para(6,3)] n(n(n(n(x) + y) + x + y + y) + n(n(x) + y)) = y.

47 (wt=29) [para(24,3)] n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + z) + n(y + z)) = z.

48 (wt=27) [para(24,3)] n(n(n(n(x) + y) + n(n(x) + y) + x + y + y) + y) = n(n(x) + y).

146 (wt=29) [para(48,3)] n(n(n(n(x) + y) + n(n(x) + y) + x + y + y + y) + n(n(x) + y)) = y.

250 (wt=34) [para(47,3)] n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + n(y + z) + z) + z) = n(y + z).

996 (wt=42) [para(250,3)] n(n(n(n(n(n(x) + y) + x + y + y) + n(n(x) + y) + n(y + z) + z) + z + u) + n(n(y + z) + u)) = u.

16379 (wt=21) [para(5,996),demod([3])] n(n(n(n(x) + x) + x + x + x) + x) = n(n(x) + x).

16387 (wt=29) [para(16379,3)] n(n(n(n(n(x) + x) + x + x + x) + x + y) + n(n(n(x) + x) + y)) = y.

16388 (wt=23) [para(16379,3)] n(n(n(n(x) + x) + x + x + x + x) + n(n(x) + x)) = x.

16393 (wt=29) [para(16388,3)] n(n(n(n(x) + x) + n(n(x) + x) + x + x + x + x) + x) = n(n(x) + x).

16426 (wt=37) [para(16393,3)] n(n(n(n(n(x) + x) + n(n(x) + x) + x + x + x + x) + x + y) + n(n(n(x) + x) + y)) = y.

17547 (wt=60) [para(146,16387)] n(n(n(n(n(x) + x) + n(n(x) + x) + x + x + x + x) + n(n(n(x) + x) + x + x + x) + x) + x) = n(n(n(x) + x) + n(n(x) + x) + x + x + x + x).

17666 (wt=33) [para(24,16426),demod([17547])] n(n(n(x) + x) + n(n(x) + x) + x + x + x + x) = n(n(n(x) + x) + x + x + x).